Whittington C of E (VA)

Primary School

CALCULATION POLICY

A guide to how we approach calculation

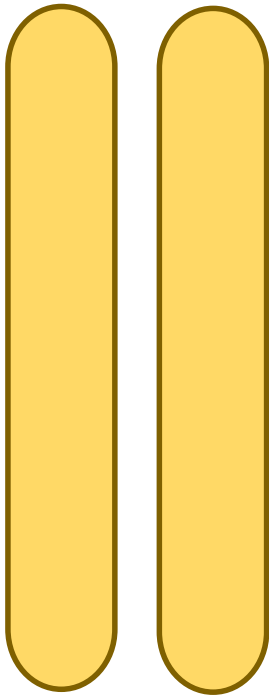
As a school we believe that all children, when introduced to a key new concept, should have the opportunity to develop their understanding by taking this approach:

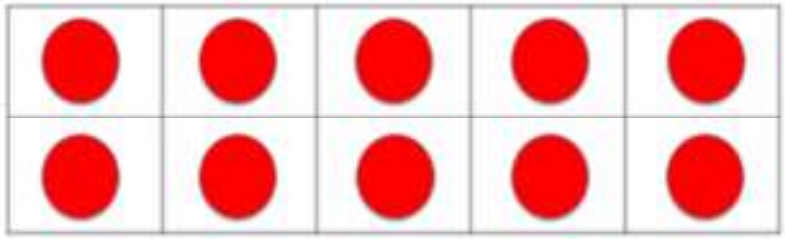
Concrete – students should have the opportunity to use concrete objects and manipulatives to help them understand what they are doing.

Pictorial – students should then build on this concrete approach by using pictorial representations. These representations can then be used to reason and solve problems.

Abstract – with the foundations firmly laid, students should be able to move to an abstract approach using numbers and key concepts with confidence.

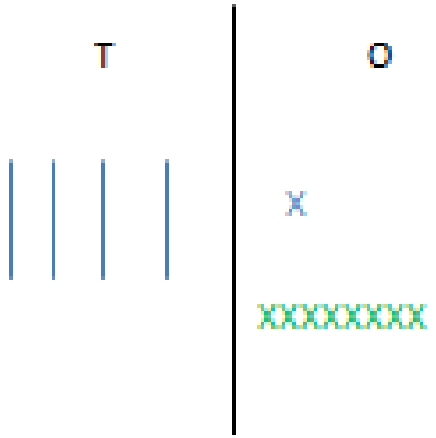
What do we mean by Concrete? The image below shows a range of resources we use that can be referred to as concrete. We also call them manipulatives or objects. They all allow children to investigate a concept practically before moving onto written work.





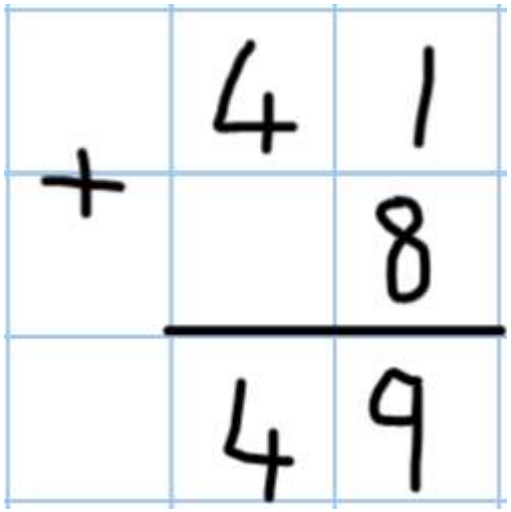
What do we mean by Pictorial?

Concrete → Pictorial



When the children have been using concrete objects and manipulatives they then move onto drawing this in their books. This is the pictorial part of our approach.

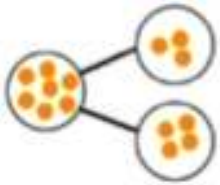
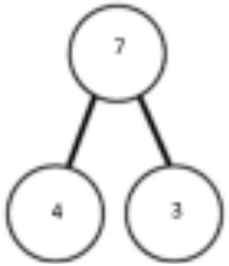
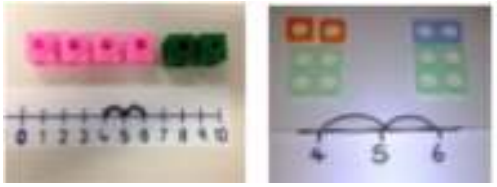
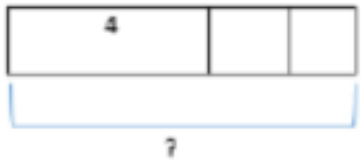
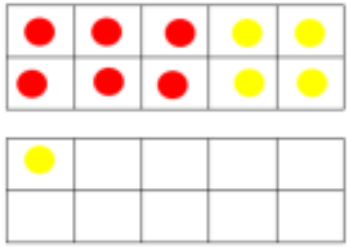
What do we mean by Abstract?

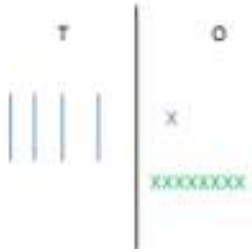
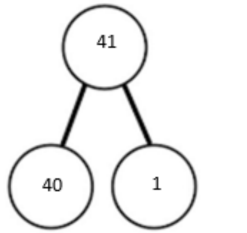
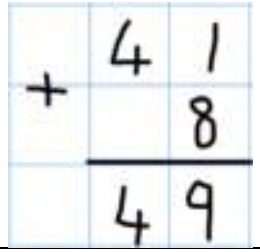
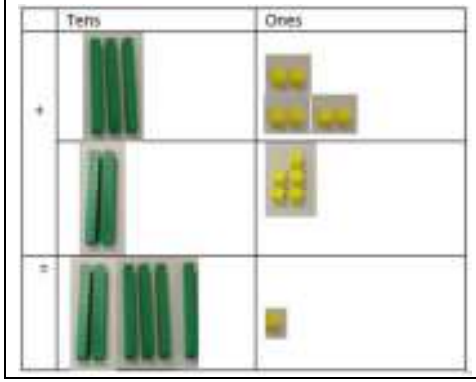
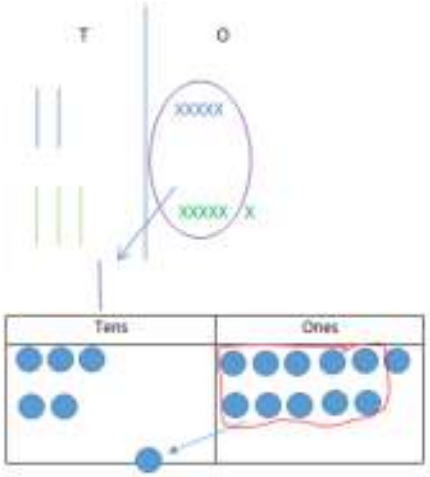
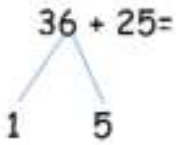


The children then move onto an abstract approach where the children are using written methods without concrete objects or pictures. This is the abstract part of our approach to calculation.

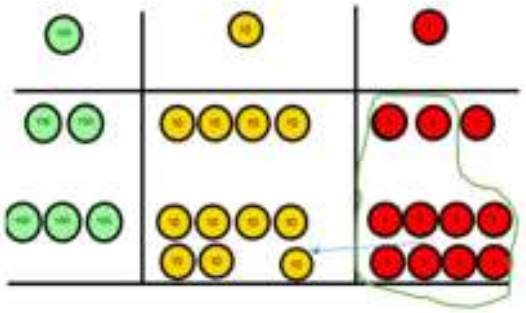
Our new calculation policy is a working document so may be periodically updated. We have based most of this policy on work done by White Rose Maths Hub, who are leaders in the field of Mastery in Mathematics. The policy is broken down into concrete, pictorial and abstract. This demonstrates how we aim to give the children a deeper understanding of calculation. The policy currently covers Year 1 through to Year 6. We will be adding more detail about our approach to number in Reception during the Summer term.

Addition- *Key language which should be used: sum, total, parts and wholes, plus, add, altogether, more than, 'is equal to' 'is the same as'*

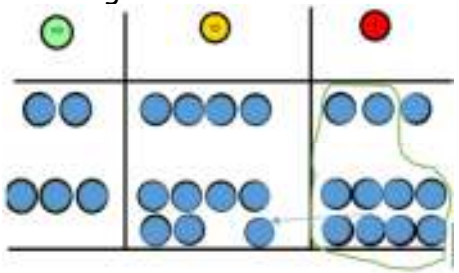
Concrete	**Pictorial**	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears etc) 	4 + 3 = 7 (four is a part, 3 is a part and the whole is seven) 	4 + 3 = 7 (four is a part, 3 is a part and the whole is seven) 
Counting on using number lines by using cubes or numicon 	A bar model which encourages the children to count on 	The abstract number line: What is 2 more than 4? What is the sum of 4 and 2? What's the total of 4 and 2? 4 + 2 
Regrouping to make 10 by using ten frames and counters/cubes or using numicon: 6 + 5 	Children to draw the ten frame and counters/cubes 	Children to develop an understanding of equality e.g 6 + ☐ = 11 and 6 + 5 = 5 + ☐ 6 + 5 = ☐ + 4

TO + O using base 10. Continue to develop understanding of partitioning and place value 41 + 8 	Children to represent the concrete using a particular symbol e.g. lines for tens and dot/crosses for ones. 	41 + 8 1 + 8 = 9 40 + 9 = 49  
TO + TO using base 10. Continue to develop understanding of partitioning and place value and usn this to support addition. Begin with no exchanging. 36 + 25 	This could be done one of two ways: 	Looking for ways to make 10 30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61  Formal method: 36 +25 61 1

Use of place value counters to add HTO + TO, HTO + HTO etc. once the children have had practice with this, they should be able to apply it to larger numbers and the abstract



Children to represent the counters e.g. like the image below

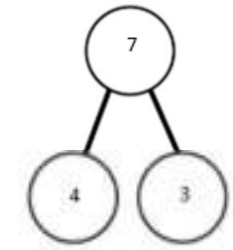


If the children are completing a word problem, draw a bar model to represent what it's asking them to do

?	
243	368

$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 1\ 1 \end{array}$$

Fluency variation, different ways to ask children to solve 21+34:



Sam saved £21 one week and £34 another. How much did he save in total?

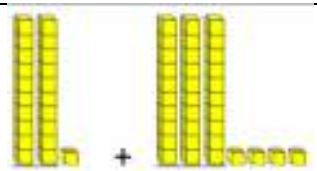
21+34=55. Prove it! (reasoning but the children need to be fluent in representing this)

$$\begin{array}{r} 21 \\ +34 \\ \hline \\ \hline \end{array}$$

21 + 34 =

□ = 21 + 34

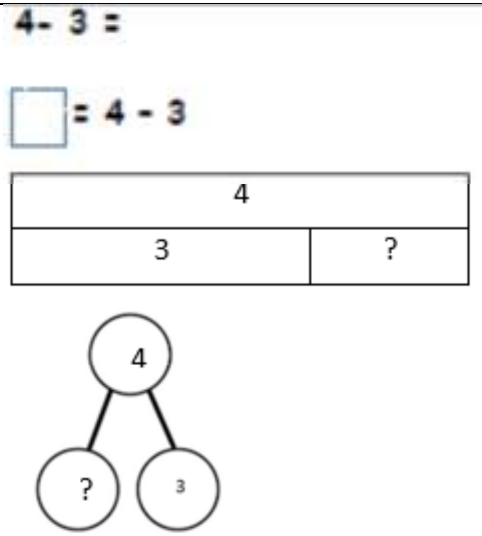
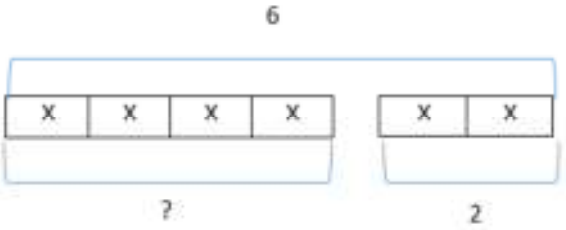
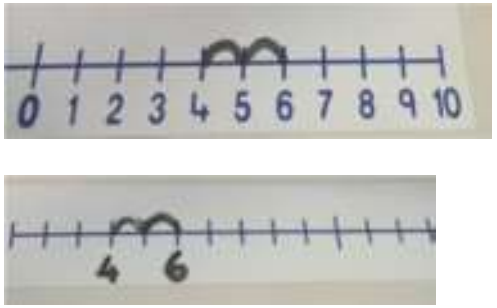
What's the sum of twenty one and thirty four?

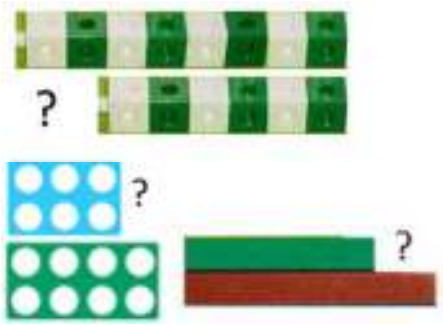
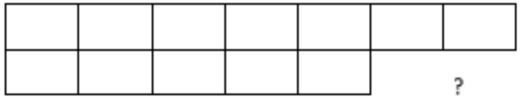
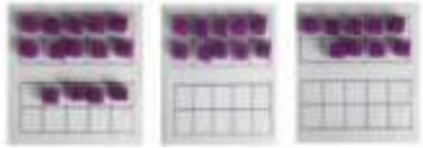
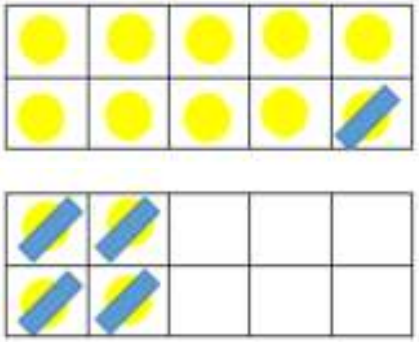
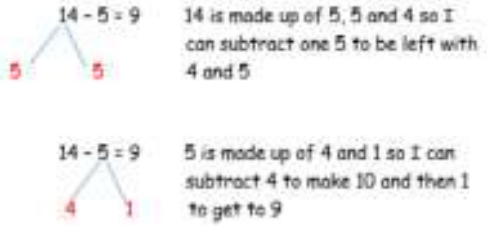
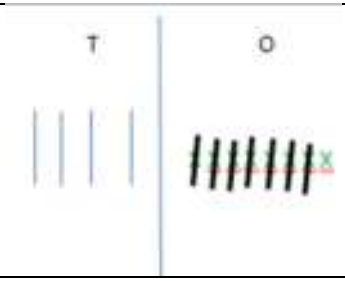
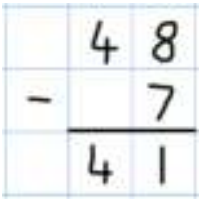


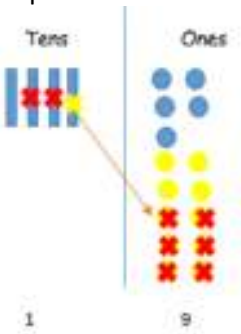
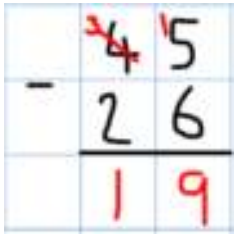
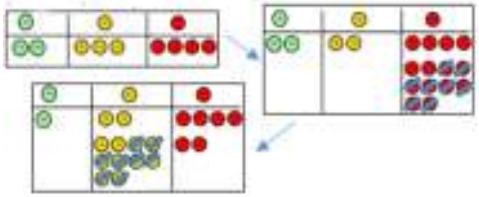
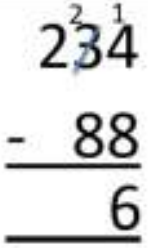
Always use missing digit problems too:

Tens	Ones
	?
?	4

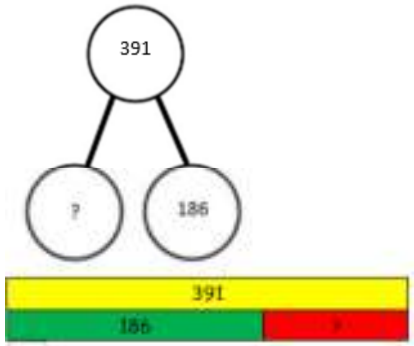
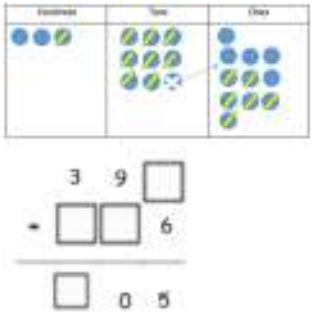
Subtraction- *Key language which should be used: take away, less than, the difference, subtract, minus, fewer, decrease, '7 take away 3, the difference is four'*

Concrete	**Pictorial**	**Abstract**
Physically taking away and removing objects from a whole (use various objects too) rather than crossing out- children will physically remove the objects 	Children to draw the concrete resources they are using and cross out.  Use of the bar model: 	
Counting back (using number lines or number tracks) 	Children to represent what they see pictorially e.g. 	

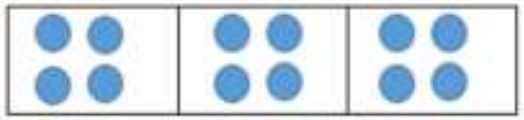
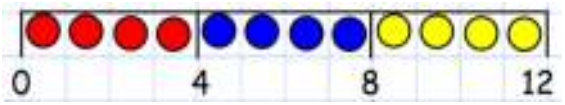
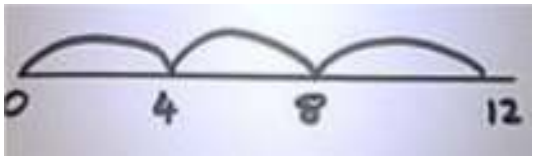
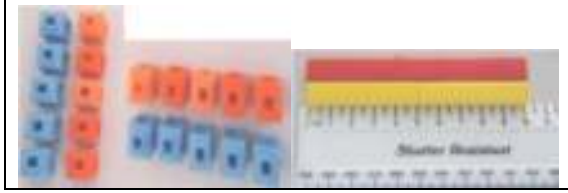
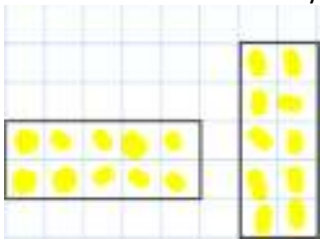
Finding the difference (using cubes, or Cuisenaire rods, other objects can also be used) 	Children to draw the cubes/other concrete objects which they have used XXXXXXXX XXXXXX Use of the bar model 	Find the difference between 8 and 6. 8 – 6, the difference is ? Children to also explore why 9 - 7 = 8 – 6 (the difference, of each digit, has changed by 1 do the difference is the same- this will help when solving 10000-9987)
Making 10 (using ten frames) 14 – 5  Children could also do this by subtracting a 5 from the 10.	Children to present the ten frame pictorially 	14 – 5 = 9 You also want children to see related facts e.g. 15 – 9 = 5 Children to represent how they have solved it e.g. 
Column method (using base 10) 48-7 		48 – 7 = 

Column method (using base 10 and having to exchange) 45-26  1) Start by partitioning 45 2) Exchange one ten for ten more ones 3) Subtract the ones, then the tens.	Represent the base 10 pictorially 	It's crucial that the children understand that when they have exchanged the 10 they still have 45. 45 = 30 + 15 
Column method (using place value counters) 234-88 	Once the children have had practice with the concrete, they should be able to apply it to any subtraction. Like the other pictorial representations, children to represent the counters.	

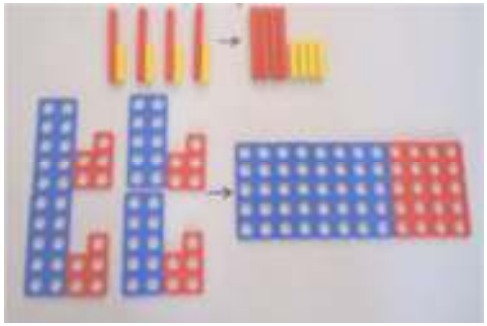
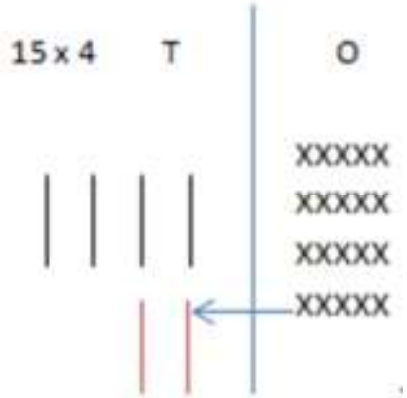
Fluency variation, different ways to ask children to solve 391-186:

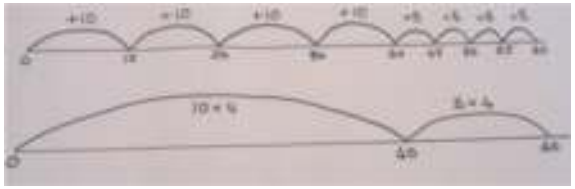
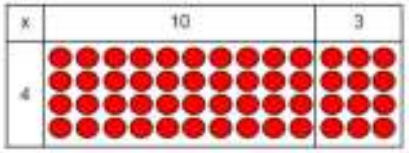
	Raj spent £391, Timmy spent £186. How much more did Raj spend? I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?	391 – 186 ☐ = 391 – 186 391 -186 ____ Find the difference between 391 and 186 Subtract 186 from 391. What is 186 less than 391?	What's the calculation? What's the answer? 

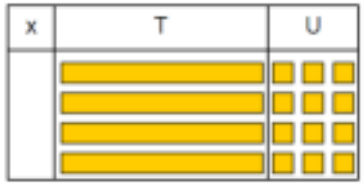
Multiplication- *Key language which should be used: double times, multiplied by, the product of, groups of, lots of, 'is equal to' 'is the same as'*

Concrete	**Pictorial**	**Abstract**
Repeated grouping/repeated addition (does not have to be restricted to cubes) 3 x 4 or 3 lots of 4 	Children to represent the practical resources in a picture e.g. XX XX XX XX XX XX Use of a bar model for a more structured method 	3 x 4 4 + 4 + 4
Use number lines to show repeated groups- 3 x 4 	Represent this pictorially alongside a number line e.g: 	Abstract number line 3 x 4 = 12 
Use arrays to illustrate commutativity (counters and other objects can also be used) 2 x 5 = 5 x 2 	Children to draw the arrays 	Children to be able to use an array to write a range of calculations e.g. 2 x 5 = 10 5 x 2 = 10 2 + 2 + 2 + 2 + 2 = 10 5 + 5 =10

<table>
<tr>
<td>Partition to multiply (use , base 10, Cuisenaire rods)

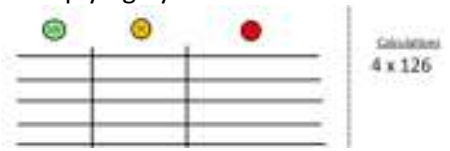
4 x 15
  </td>
<td>Children to represent the concrete manipulatives in a picture e.g. base 10 can be represented like:
 
</td>
<td>Children to be encouraged to show the steps they have taken
4 x 15
10 5

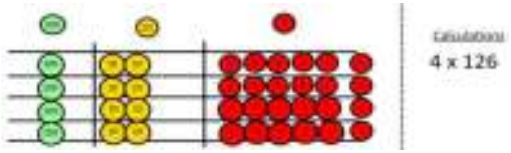
10 x 4 = 40
5 x 4 = 20
40 + 20 = 60
A number line can also be used
  </td>
</tr>
<tr>
<td>Show the link with arrays to first introduce the grid method.
 

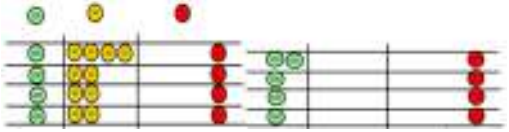
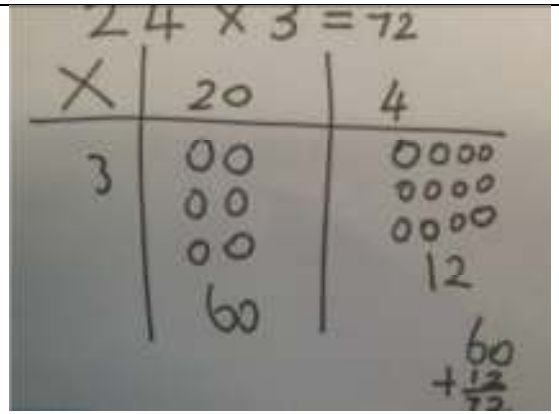
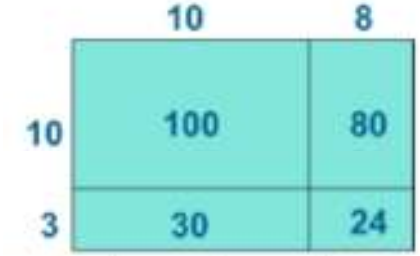
Move on to using Base 10 to move towards a more compact method.
 
</td>
<td>Children can represent the work they have done with place value counters in a way that they understand.
They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.</td>
<td>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.
 

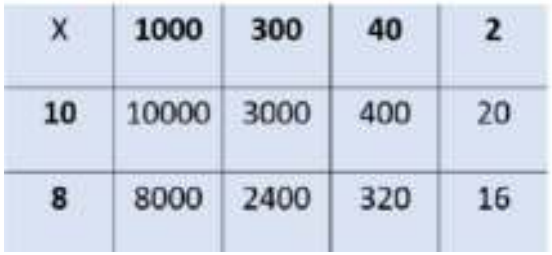
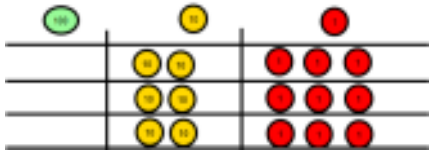
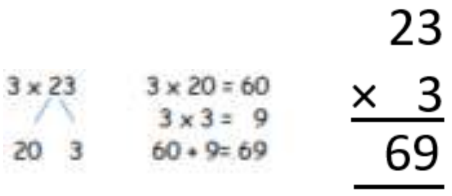
210 + 35 = 245

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</td>
</tr>
</table>

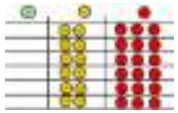
<table>
<tr>
<td>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.
 

Fill each row with 126.
 

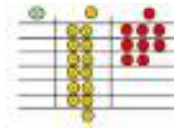
Add up each column, starting with the ones making any exchanges needed.
 
Then you have your answer.</td>
<td> 
</td>
<td> 

 
</td>
</tr>
<tr>
<td>Formal column method with place value counters or base 10 (at the first stage- no exchanging) 3 x 23
Make 23, 3 times. See how many ones, then how many tens
  </td>
<td>Children to represent the counters in a pictorial way
 
</td>
<td>Children to record what it is they are doing to show understanding
 
</td>
</tr>
</table>

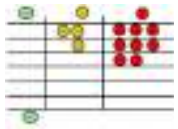
Formal column method with place value counters (children need this stage, initially, to understand how the column method works)



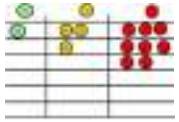
6 x 23
Step 1: get 6 lots of 23



Step 2: 6 x 3 is 18. Can I make an exchange? Yes! Ten ones for one ten….

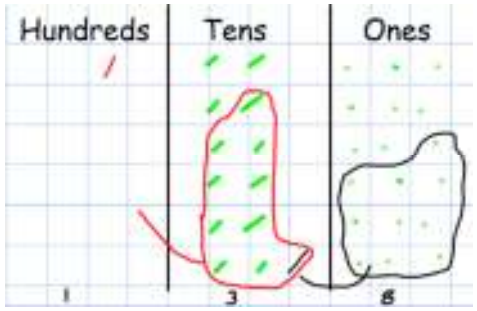


Step 3: 6 x 2 tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred…



Step 4- what do I have I each column?

Children to represent the counters/base 10, pictorially e.g. the image below.



6 x 23
6 x 3 = 18
6 x 20 = 120
120 + 18 = 138

The aim is to get to the formal method but the children need to understand how it works.

$$\begin{array}{r} 6 \times 23 = \\ \\ 23 \\ \times\ \ 6 \\ \hline 138 \\ \hline \end{array}$$

1 1

When children start to multiply 3d x 3d and 4d x 2d etc, they should be confident with the abstract:
To get 744 children have solved 6 x 124
To get 2480 they have solved 20 x 124

$$\begin{array}{rrrr} & 1 & 2 & 4 \\ \times & & 2 & 6 \\ \hline & {}^{1}7 & {}^{2}4 & 4 \\ 2 & {}^{1}4 & 8 & 0 \\ \hline 3 & 2 & 2 & 4 \\ \hline \end{array}$$

1 1

Fluency variation, different ways to ask children to solve 6 x 23:

With the counters, prove that 6 x 23 = 138
Why is 6 x 23 =23 x 6?

Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

Tom saved 23p three days a week. How much did he save in 2 weeks?

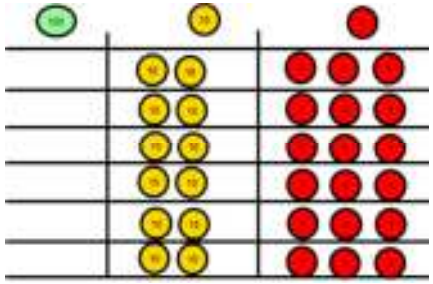
Find the product of 6 and 23
6 x 23 =

$$\begin{array}{r} 6 \\ \times\ 23 \\ \hline \\ \hline \end{array}$$

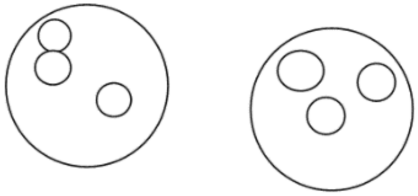
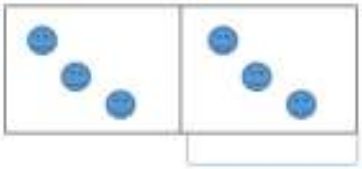
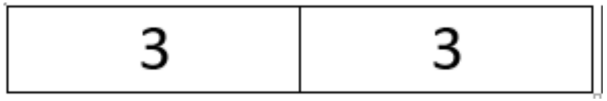
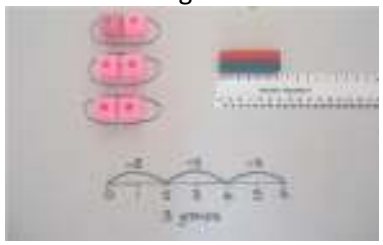
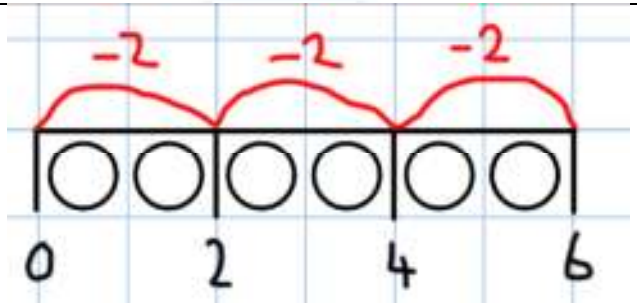
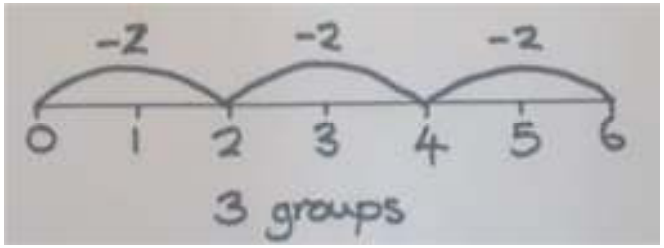
$$\begin{array}{r} 23 \\ \times\ 6 \\ \hline \\ \hline \end{array}$$

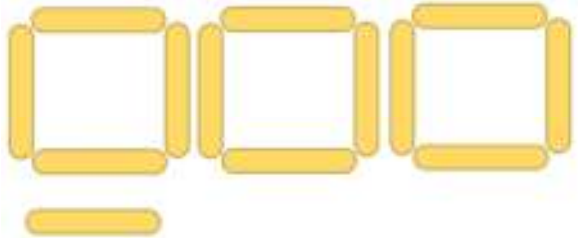
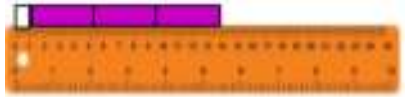
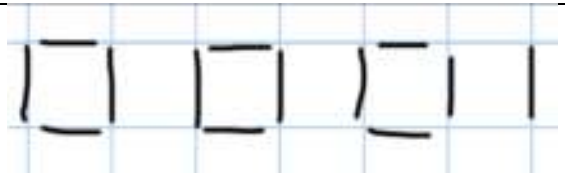
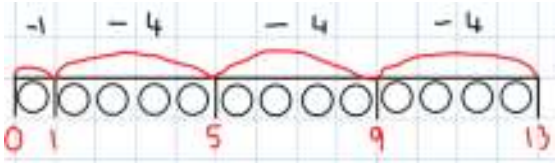
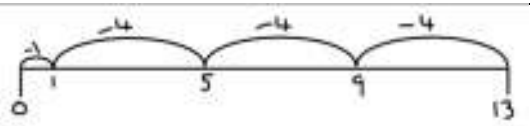
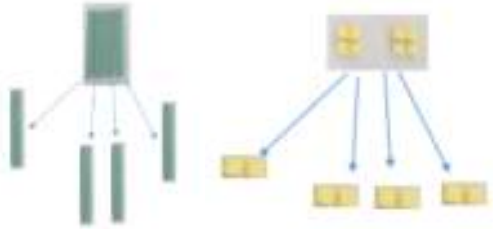
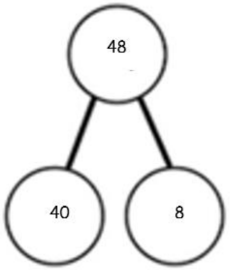
$\square = 6 \times 23$

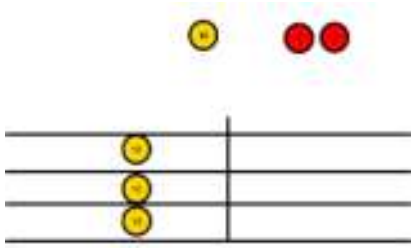
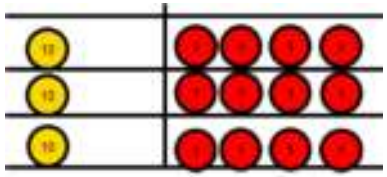
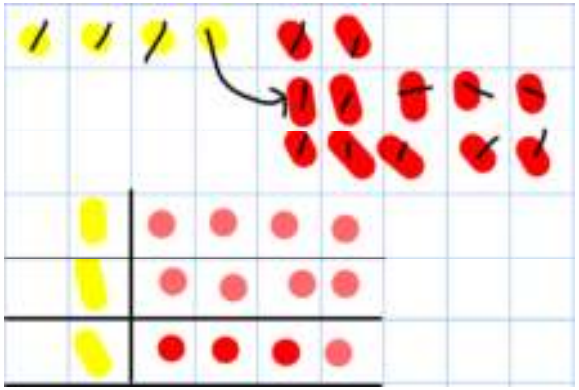
What's the calculation? What's the answer?

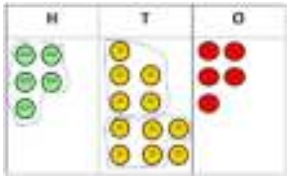
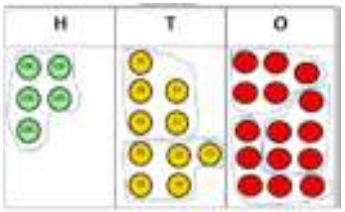
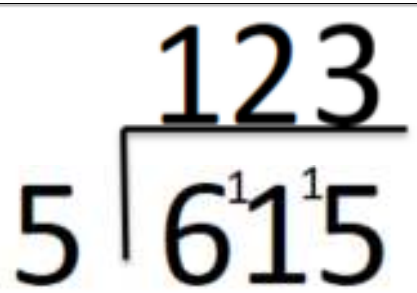


Division- *Key language which should be used: share, group, divide, divided by, half, 'is equal to' 'is the same as'*

Concrete	**Pictorial**	**Abstract**
6 shared between 2 (other concrete objects can also be used e.g. children and hoops, teddy bears, cakes and plates) 	 This can also be done in a bar so all 4 operations have a similar structure: 	6 ÷ 2 = 3 What's the calculation? 
Understand division as repeated grouping and subtracting 6 ÷ 2 		Abstract number line 
2d ÷ 1d with remainders 13 ÷ 4 – 3 remainder 1	Children to have chance to represent the resources they use in a pictorial way e.g. see below:	13 ÷ 4 = 3 remainder 1 Children to count their times tables facts in their heads

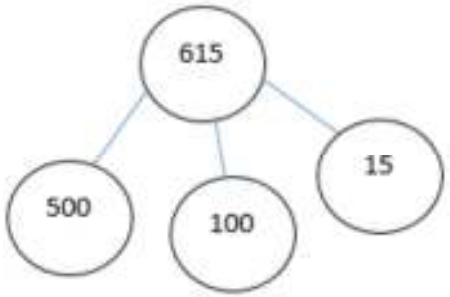
Use of lollipop sticks to form wholes  Use of Cuisenaire rods and rulers (using repeated subtraction) 	 	
2d divided by 1d using base 10 (no remainders) SHARING 48 ÷ 4 = 12 	Children to represent the base 10 and sharing pictorially.	48 ÷ 4  4 tens ÷ 4 = 1 ten 8 ones ÷ 4 = 2 ones 10 + 2 = 12

Sharing using place value counters. 42 ÷ 3= 14  1. Make 42. Share the 4 tens between 3. Can we make an exchange with the extra 10?  Exchange the ten for 10 ones and share out 12 ones		42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14
Use base 10 materials to demonstrate chunking	Draw the base ten materials	Q. 537 div. by 6 Example: 537 300 - 50 x 6 237 180 - 30 x 6 57 54 - 9 x 6 3 50+30+9 = 89 Ans: 89 rem 3

Use of the 'bus stop method' using grouping and counters. Key language for grouping- how many groups of X can we make with X hundreds'- this can also be done using sharing! 615 ÷ 5  Step 1: make 615  Step 2: Circle your groups of 5  Step 3: Exchange 1H for 10T and circle groups of 5  Step 4: exchange 1T for 10ones and circles groups of 5	This can easily be represented pictorially, till the children no longer to do it. It can also be done to decimal places if you have a remainder!	

Fluency variation, different ways to ask children to solve 615 ÷ 5:

Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method?

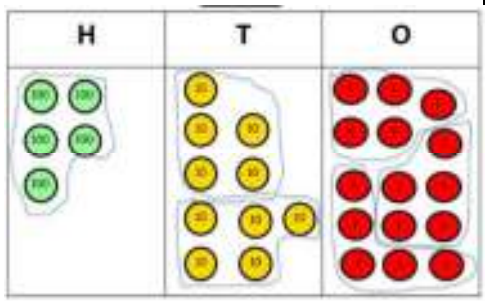


I have £615 and share it equally between 5 bank accounts. How much will be in each account?
615 pupils need to be put into 5 groups. How many will be in each group?

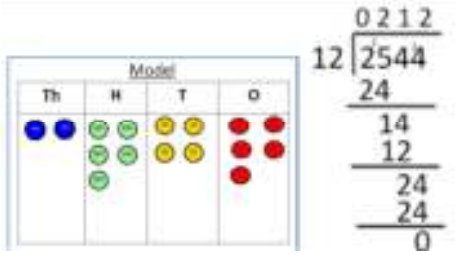
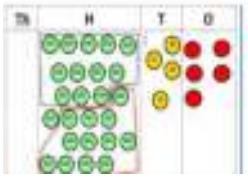
$5\overline{)615}$

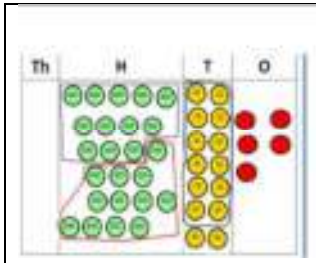
615 ÷ 5 =
= 615 ÷ 5
How many 5's go into 615?

What's the calculation? What's the answer?



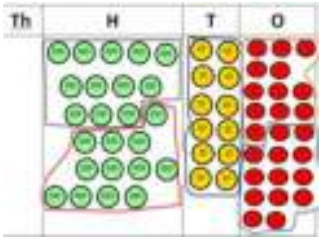
Long Division

Concrete	Pictorial	Abstract
 0212 12 2544 24 14 12 24 24 0 2544 ÷ 12 How many groups of 12 thousands do we have? None  Exchange 2 thousand for 20 hundreds.  02 12 2544 24 1 How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one.	Children to represent the counters, pictorially and record the subtractions beneath.	0 12 2544 Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds. 02 12 2544 24 1 Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left. 021 12 2544 24 14 12 2 Exchange the one hundred for 10 tens. How many



$$\begin{array}{r} 021 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.



Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2

groups of 12 can I make with 14 tens?
The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left

$$\begin{array}{r} 0212 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.